Circular Base Plates with Large Eccentric Loads

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Abstract: Circular base plates are commonly used for pipe columns, such as pylons in cable-stayed bridges, lighting poles, and electric power line posts. Although the explicit solutions for rectangular base plates can be found in many textbooks and AISC design procedures for base plates, a design procedure for circular base plates has not been documented. In this paper, two equilibrium equations are presented for circular base plates with large eccentric loads. Since these two equations cannot be solved explicitly, an iteration approach is used to solve them.

DOI: 10.1061/(ASCE)1084-0680(2004)9:3(142)

CE Database subject headings: Base plates; Eccentric loads; Equilibrium equations.

Introduction

Base plates are usually used to distribute column loads to a supporting concrete foundation. Depending on the column cross sections, base plates can be rectangular or circular shapes. Rectangular base plates are obviously suitable for steel columns with 1 or W sections. Circular base plates are commonly used for pipe columns, such as pylons in cable-stayed bridges, lighting poles, and electric power line posts.

Several different loading conditions are considered for the design of base plates. Under axial load, the bearing pressure is uniformly distributed between the base plate and the supporting concrete. The base plate size and thickness can be easily determined based on the allowable concrete-bearing capacity and design-bearing stress. A minimum number of anchor bolts should be provided.

When the axial load is combined with moment, base plates experience small, moderate, and large eccentricities which equal to the moment divided by the axial force. For small and moderate eccentricities, the bearing stress occurs on the full or partial base plate, respectively. A linear bearing stress distribution is usually assumed. When ASD (allowable stress design) is used, the maximum bearing stress must not exceed the allowable bearing stress. The resultant for the bearing stress must be equal to the axial load. A minimum number of anchor bolts should be provided in this case as well.

For large eccentricity, the compressive bearing area is less than a half of total base plate area, and it is necessary to provide enough anchor bolts to resist the tensile component resulting from the moment. The design procedure for rectangular base plates with large eccentricity can be easily found in many textbooks and AISC (1989, 1990) steel design guide series. However, the design procedure for circular base plates has not been documented. In this paper, two equilibrium equations are presented for circular base plates with large eccentric loads. Since these two equations cannot be solved explicitly, an iteration approach is used to solve these two equations. A step-by-step ASD procedure is developed and a design example is given.

Analysis of Circular Base Plates with Large Eccentric Loads

The following assumptions are used to analyze circular base plates with large eccentric loads:

1. Elastic behavior.
2. The maximum bearing stress is equal to the allowable value.
3. The compressive bearing area is less than half of the circular base plate (large eccentricities).
4. The resultant compressive bearing stress is located at the c.g. (center of gravity) of the compressive bearing area. The verification of this assumption is shown in the Appendix.
5. Only the anchor bolts in the other half of the circle are considered in tension. This is a simplified and conservative assumption.
6. The critical section used to determine the base plate thickness should be based on 0.80 times the outside dimension of round columns. If stiffeners are provided, the critical section may be based on 1.0 times the outside dimension of round columns.

Fig. 1 shows the plan and elevation views of the circular base plate under large eccentric loads. Two equilibrium equations can be established to determine the unknowns, such as the magnitude of the resultant anchor bolt force T and the length of the bearing A. The sum of the forces yields

$$P+T=F_p\frac{C}{A}A_{\mathrm{seg}} \tag{1}$$

It is noticed that the right side of Eq. (1) is the resultant compressive bearing stress, and this formula is verified in the Appendix.

The sum of moments about the resultant bolt force yields

$$P(e+A')=F_p\frac{C}{A}A_{\mathrm{seg}}\left[\frac{N}{2}-(A-C)+A'\right] \tag{2}$$

where e=eccentricity, equal to the moment M divided by the axial force P; N=diameter of the circular base plate; A=rise of the circular segment (compressive stress bearing area); A'=distance

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Note. Discussion open until January 1, 2005. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on October 22, 2002; approved on December 4, 2002. This paper is part of the ***Practice Periodical on Structural Design and Construction***, Vol. 9, No. 3, August 1, 2004. ©ASCE, ISSN 1084-0680/2004/3-142–146/$18.00.

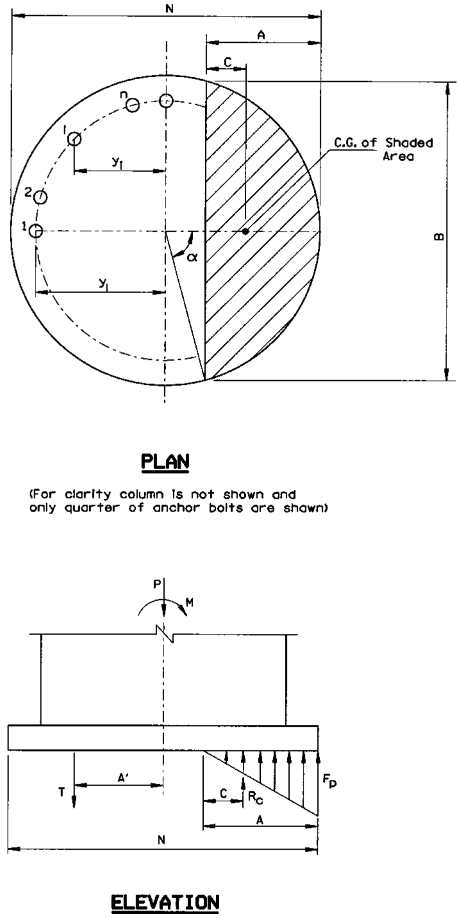

Fig. 1. Circular base plates with large eccentric loads

between the c.g. of the anchor bolt forces and the column center; A_{seg}=compressive bearing area; C=distance from the c.g. of compressive stress bearing area to the zero stress section; F_p=allowable bearing stress; and T=sum of the anchor bolt forces in the half of the circle.

The maximum anchor bolt force can be determined by summing the anchor bolt forces:

$$T_1+2\sum_{n=2}^{n} T_i=T \tag{3}$$

where T_1=outmost anchor bolt force; and T_i=ith anchor bolt force. Since the strain distribution is linear, we have

$$T_1=T_{max} \tag{4}$$

and

$$\varepsilon_i=\frac{y_i}{y_1}\varepsilon_1 \tag{5}$$

where y_i=distance from the ith anchor bolt to the centerline of the base plate; y_1=distance from the outmost anchor bolt to the centerline of the base plate; ε_i=strain at the ith anchor bolt; and ε_1=strain at the outmost anchor bolt.

The ith anchor bolt force, T_i, can be presented as follows:

$$T_i=\sigma_i A_{bolt}=E\varepsilon_i A_{bolt}=E\frac{y_i}{y_1}\varepsilon_1 A_{bolt} \tag{6}$$

where σ_i=ith anchor bolt stress; E=modulus of elasticity of steel; and A_{bolt}=area of each anchor bolt.

Substituting Eq. (6) into Eq. (3) yields

$$T_1=T_{max}=\frac{T}{1+\frac{2}{y_1}\sum_{n=2}^{n} y_i} \tag{7}$$

Since Eqs. (1) and (2) cannot be solved explicitly, an iteration approach is used to solve them. A design procedure for AISC-ASD to analyze the circular base plates with large eccentric loads is proposed as follows:

1. Determine the maximum allowable stress (AISC 1990),

$$F_p=0.35f'_c\sqrt{\frac{A_2}{A_1}}\leqslant 0.7f'_c \tag{8}$$

2. Assume a trial base plate size, N.
3. Assume total number of the anchor bolts and their diameter.
4. Determine A', the c.g. of the anchor bolts in the half of the circle.
   Pick a trial-bearing length A.
   Determine the section properties of the circular segment (compressive bearing area) as follows:

$$\alpha=\arccos\left(\frac{N/2-A}{N/2}\right)$$

$$B=2(N/2)\sin\alpha$$

   Determine the area of the segment (AISC 1989):

$$A_{seg}=0.0087266\left(\frac{N}{2}\right)^2(2\alpha)-B\left(\frac{N}{2}-A\right)\Big/2$$

   (α is in degrees as shown in Fig. 1.)
   Determine the c.g. of the segment (Young 1989):

$$\text{If}\quad \alpha\geqslant\pi/4,\quad C=\frac{N}{2}\left[\frac{2\sin^3\alpha}{3(\alpha-\sin\alpha\cos\alpha)}-\cos\alpha\right]$$

$$\text{If}\quad \alpha<\pi/4,\quad C=0.2\frac{N}{2}\alpha^2(1-0.0619\alpha^2+0.0027\alpha^4)$$

5. Determine the values of both the left and right sides of Eq. (2). If the value of the left side is equal to that of the right side, go to Step 6. Otherwise return to Step 4.
6. Determine the resultant anchor bolt force T from Eq. (1).
7. Determine T_{max} from Eq. (7). If T_{max}<allowable anchor bolt load, go to Step 8. Otherwise return to Step 2 or 3.
8. Determine the base plate thickness, based on the elastic bearing stress distribution:

$$t_p=\sqrt{\frac{6M_{pl}}{F_b}} \tag{9}$$

   where M_{pl}=moment for a 1 mm (or 1 in.) wide strip at the critical section, i.e., the total moment at the critical section divided by the chord at the critical section; and F_b is the allowable bending stress, equal to $0.75F_y$.

This is a trial-and-error iteration that is easily accomplished using a spreadsheet.

Example

Design a circular base plate for an axial load of 200 kips (889.6 kN) and a moment of 20,000 in-kips (2260 m-kN), as shown in Fig. 2. The outside diameter of the steel column pipe is 1067 mm (42 in.). The ratio of concrete to plate areas A_2/A_1=1.5. The allowable stresses for the anchor bolts and the base plate are 44

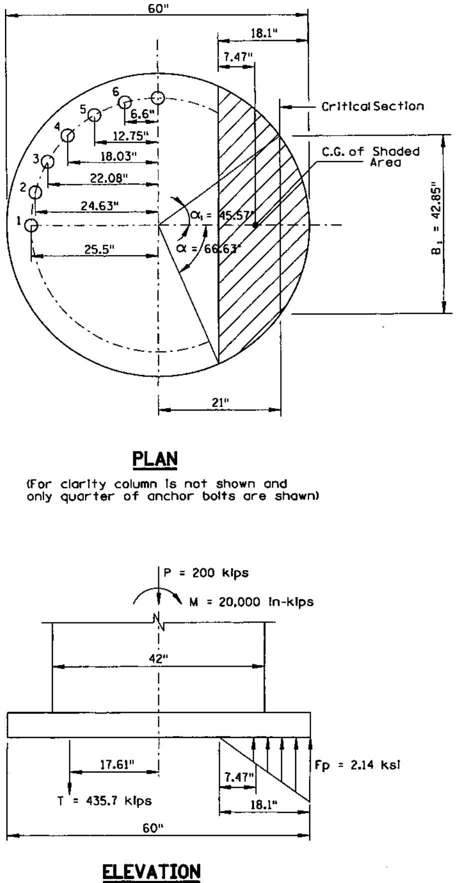

Fig. 2. Design example (1 ft-kip=1.356 m-kN, 1 ft=0.3048 m, 1 ksi=6.895 MPa)

ksi (303 MPa) and 50 ksi (345 MPa), respectively. The concrete strength f'_c is 5 ksi (35 MPa). The allowable stress design is used in this design example.

1. The maximum design bearing stress is

$$F_p = 0.35(5)\sqrt{1.5} = 2.14 \text{ ksi}$$

2. Assume $N = 60$ in.

$$e = \frac{M}{P} = \frac{20{,}000}{200} = 100 \text{ in.} > \frac{N}{2} = \frac{60}{2} = 30 \text{ in.}$$

This is the case for large eccentricity.

3. Assume total number of the anchor bolts=24 and their diameter=1.5 in. The distance from the c.g. of the anchor bolts in the half of the circle to the centerline of the base plate (Fig. 2) is

$$A' = [25.50 + 2(24.63 + 22.08 + 18.03 + 12.75 + 6.60)]/11$$

$$= 17.61 \text{ in.}$$

4. Assume $A = 18.1$ in.

$$\alpha = (\pi/180)[\arccos(30 - 18.1)/30] = 1.163 \text{ radians} = 66.63°$$

$$B = 2(60/2)\sin 66.63 = 55.08 \text{ in.}$$

and the area of the segment (compressive bearing area):

$$A_{\text{seg}} = 0.0087266(60/2)^2(2)(66.63) - 55.08(60/2 - 18.1)/2$$

$$= 719 \text{ in.}^2$$

Since $\alpha = 1.163 \geqslant \pi/4 = 0.785$, the c.g. of the segment is

$$C = \frac{60}{2}\left[\frac{2\sin^3 66.63}{3(1.163 - \sin 66.63\cos 66.63)} - \cos 66.63\right]$$

$$= 7.47 \text{ in.}$$

5. Determine both left and right sides of Eq. (2):

$$P(e + A') = 200(100 + 17.61) = 23{,}522 \text{ in-kips}$$

$$F_p\frac{C}{A}A_{\text{seg}}\left[\frac{N}{2} - (A - C) + A'\right]$$

$$= 2.14\left(\frac{7.47}{18.1}\right)(719)\left[\frac{60}{2} - (18.1 - 7.47) + 17.61\right]$$

$$= 23{,}506 \text{ in-kips}$$

The difference between the left and right sides of Eq. (2) is only 0.07%. Eq. (2) is satisfied.

6. Determine T from Eq. (1):

$$T = F_p\frac{C}{A}A_{\text{seg}} - P = 2.14\left(\frac{7.47}{18.1}\right)(719) - 200 = 435.7 \text{ kips}$$

7. Determine $T_{\max}$ from Eq. (7):

$$T_{\max} = \frac{435.7}{1 + \left(\frac{2}{25.50}\right)(24.63 + 22.08 + 18.03 + 12.75 + 6.60)}$$

$$= 57.36 \text{ kips}$$

The allowable anchor bolt force $= \pi/4(1.5)^2(44) = 77.75$ kips > 57.36 kips O.K.

8. Assuming the stiffeners are provided, the critical section is located at 21 in. from the centerline of the base plate. Similar to the previous calculation, consider the circular segment at the critical section and calculate the section properties of the segment as follows:

$$\alpha_1 = (\pi/180)(\arccos 21/30) = 0.795 \text{ radians} = 45.57°$$

The chord of the segment is

$$B_1 = 2(30)\sin(45.57) = 42.85 \text{ in.}$$

and the area of the segment is

$$A_{\text{seg1}} = 0.0087266(60/2)^2(2)(45.57) - 42.85(21)/2 = 266 \text{ in.}^2$$

Since $\alpha = 0.795 \geqslant \pi/4 = 0.785$, the c.g. of the segment is

$$C_1 = \frac{60}{2}\left[\frac{2\sin^3 45.57}{3(0.795 - \sin 45.57\cos 45.57)} - \cos 45.57\right] = 3.65 \text{ in.}$$

The moment at the critical section is

$$M_{pl} = \left[\frac{A - (N/2 - d/2) + C_1}{A}\right]\left(\frac{F_p A_{\text{seg1}} C_1}{B_1}\right)$$

where d=diameter of the column,

$$M_{pl} = \frac{18.1 - (60/2 - 42/2) + 3.65}{18.1}(2.14)(266)(3.65)/42.85$$

$$= 34.21 \text{ in-kips}$$

and the plate thickness is determined by the following:

$$t_p = \sqrt{\frac{6(34.21)}{0.75(50)}} = 2.34 \text{ in.}, \quad \text{use } 2.5 \text{ in.}$$

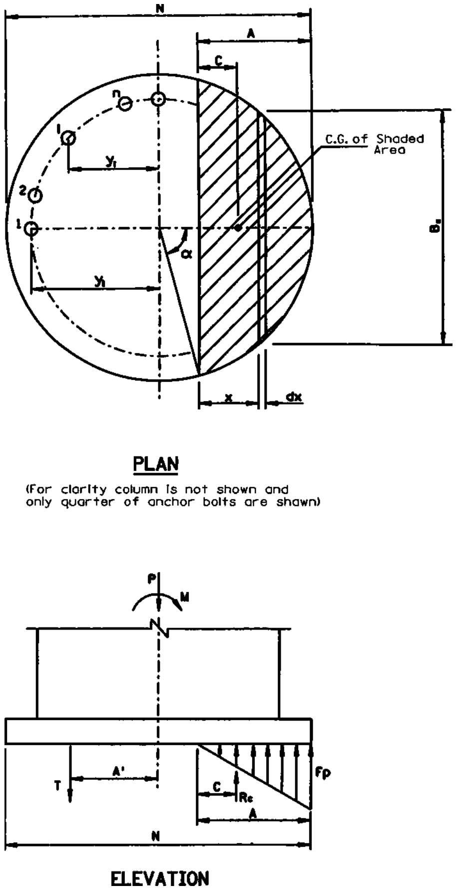

Fig. 3. Verification of resultant compressive stress in circular segment

Conclusions

Two equilibrium equations are presented for circular base plates with large eccentric loads. A detailed iteration approach is proposed to solve these two equations. A design example is given to show the implementation of this approach. Since the design procedure presented in this paper is a simplified approach, a more complicated and more accurate approach will be developed in the future.

Appendix. Verification of Resultant Compressive Stress in a Circular Segment

The resultant compressive force R_c in a circular segment, as shown in Fig. 3, can be expressed as follows:

$$R_c = F_p \frac{C}{A} A_{\text{seg}} \tag{10}$$

This is based on the assumption in which the resultant compressive bearing force is located at the c.g. of the compressive bearing area and the stress distribution is linear.

To verify Eq. (10), consider a strip of the compressive bearing force:

$$\frac{x}{A} F_p B_x dx \tag{11}$$

where x, A, F_p, B_x, and dx are shown in Fig. 3.

The resultant compressive bearing force is taken as the integral of Eq. (11), which is

$$R_c = \int_0^A \frac{x}{A} F_p B_x dx \tag{12}$$

Letting $R = N/2$, the chord can be expressed as

$$B_x = 2\sqrt{R^2 - (R - A + x)^2} \tag{13}$$

Let $y = R - A + x$ and substitute Eq. (13) into Eq. (12):

$$\begin{aligned} R_c &= \frac{2F_p}{A} \int_0^A x\sqrt{R^2 - (R-A+x)^2} dx \\ &= \frac{2F_p}{A} \int_{R-A}^{R} [y - (R-A)]\sqrt{R^2 - y^2} dy \\ &= \frac{2F_p}{A} \left[\int_{R-A}^{R} y\sqrt{R^2-y^2} dy - (R-A) \int_{R-A}^{R} \sqrt{R^2-y^2} dy \right] \end{aligned} \tag{14}$$

The solutions of the integration in Eq. (14) are given as follows (Swokowski 1983):

$$\begin{aligned} \int_{R-A}^{R} y\sqrt{R^2-y^2} dy &= -\frac{1}{3}(R^2-y^2)^{3/2} \Big|_{y=R-A}^{y=R} \\ &= \frac{1}{3}[R^2-(R-A)^2]^{3/2} \end{aligned} \tag{15}$$

and

$$\begin{aligned} \int_{R-A}^{R} \sqrt{R^2-y^2} dy &= \left[\frac{y}{2}\sqrt{R^2-y^2} + \frac{R^2}{2} \arcsin\frac{y}{R} \right] \Big|_{y=R-A}^{y=A} \\ &= \frac{\pi}{4}R^2 - \frac{R-A}{2}\sqrt{R^2-(R-A)^2} \\ &\quad - \frac{R^2}{2}\arcsin\frac{R-A}{R} \end{aligned} \tag{16}$$

Therefore, the resultant compressive bearing force is

$$\begin{aligned} R_c = \frac{2F_p}{A} \bigg\{ & \frac{1}{3}[R^2-(R-A)^2]^{3/2} - \frac{\pi}{4}R^2(R-A) \\ & + \frac{(R-A)^2}{2}\sqrt{R^2-(R-A)^2} + \frac{(R-A)R^2}{2}\arcsin\frac{R-A}{R} \bigg\} \end{aligned} \tag{17}$$

The following parameters from the design example are used to compare the values for Eqs. (10) and (17): $A = 18.1$ in., $C = 7.47$ in., $F_p = 2.14$ ksi, $A_{\text{seg}} = 719$ in.^{2}, $R = N/2 = 60/2 = 30$ in. Substituting the above design parameters into Eqs. (10) and (17) yields

$$R_c = F_p \frac{C}{A} A_{\text{seg}} = 2.14\left(\frac{7.47}{18.1}\right)(719) = 635.02 \text{ kips}$$

and

$$
\begin{aligned}
R_c = \frac{2(2.14)}{18.1} \Bigg\{ & \frac{1}{3}[30^2-(30-18.1)^2]^{3/2} - \frac{\pi}{4} 30^2(30-18.1) \\
& + \frac{(30-18.1)^2}{2}\sqrt{30^2-(30-18.1)^2} \\
& + \frac{(30-18.1)(30)^2}{2} \arcsin \frac{30-18.1}{30} \Bigg\} \\
= 634.72 \ \text{kips}
\end{aligned}
$$

The difference between the values of Eq. (10) and Eq. (17) is only 0.047%. Therefore, it is sufficiently accurate to use Eq. (10) to calculate the resultant compressive stress in a circular segment.

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